A modified naturalness principle and its experimental tests

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Abstract

Motivated by LHC results, we modify the usual criterion for naturalness by ignoring the uncomputable power divergences. The Standard Model satisfies the modified criterion ('finite naturalness') for the measured values of its parameters. Extensions of the SM motivated by observations (Dark Matter, neutrino masses, the strong CP problem, vacuum instability, inflation) satisfy finite naturalness in special ranges of their parameter spaces which often imply new particles below a few TeV. Finite naturalness bounds are weaker than usual naturalness bounds because any new particle with SM gauge interactions gives a finite contribution to the Higgs mass at two loop order.

Contents

1 Introduction

The naturalness principle strongly influenced high-energy physics in the past decades [1], leading to the belief that physics beyond the Standard Model (SM) must exist at a scale $\Lambda_{\rm NP}$ such that quadratically divergent quantum corrections to the Higgs squared mass are made finite (presumably up to a log divergence) and not much larger than the Higgs mass m_h itself. This ideology started to conflict with data after TeVatron measured the top mass (which implies a sizeable order-one top Yukawa coupling λ_t) and after LEP excluded new charged particles below 100 GeV [2]. Indeed, imposing that the SM one loop correction to m_h^2

$$\delta m_h^2 \approx \delta m_h^2(\text{top loop}) \approx \frac{12\lambda_t^2}{(4\pi)^2}\Lambda_{\rm NP}^2 \times \left\{ \begin{array}{l} 1 \\ \ln M_{\rm Pl}^2/\Lambda_{\rm NP}^2 \end{array} \right. \tag{1}$$

is smaller than $m_h^2 \times \Delta$, where Δ is the amount of allowed fine-tuning (ideally $\Delta \lesssim 1$), implies

$$\Lambda_{\rm NP} \lesssim \sqrt{\Delta} \times \left\{ \begin{array}{l} 400\,\text{GeV} \\ 50\,\text{GeV} \end{array} \right. . \tag{2}$$

The most plausible new physics motivated by naturalness is supersymmetry. It adds new particles at the weak scale, with the lightest one possibly being Dark Matter (DM). Further support for this scenario come from gauge unification and from the fact that DM loosely around the weak scale is indicated by the hypothesis that DM is the thermal relic of a massive stable particle. Doubting that nature is natural seemed impossible.

However, no new physics has been so far seen at LHC with $\sqrt{s} = 8\,\text{TeV}$, such that in most models the unit of measure for Δ presently is the kilo-fine-tuning. While this is not conclusive evidence, while naturalness arguments can be weakened by allowing for a finer tuning, while various searches have not yet been performed, while LHC will run at higher energy, etc, it is fair to say that the most straightforward interpretation of present data is that the naturalness ideology is wrong.

This situation lead to consider the opposite extremum: the Higgs is light due to huge cancelations [3] because 'anthropic selection' destroyed naturalness.

Here we explore an intermediate possibility, that sometimes appeared in the literature, more or less implicitly. We name it *'finite naturalness'*. The idea is that we should ignore the uncomputable quadratic divergences, so that the Higgs mass is naturally small as long as there are no heavier particles that give large finite contributions to the Higgs mass.

Some authors explored the possibility that quadratic divergences vanish because the true physical cut-off behaves like dimensional regularization. For example, this might maybe happen in special quantum gravity models where the Planck scale arises from the breaking of a dilatation-like symmetry [4]. Another possibility is an infinite tower of states at the Planck scale, arranged in a way that cancels power divergences [5]. Alternatively, new physics might allow for a Veltman throat at the Planck scale (a possibility which has been excluded within the SM [6]). These speculations are not based on theoretically firm grounds.

Anyhow, the goal of this paper is not advocating for the 'finite naturalness' scenario.

Instead, we want to explore how experiments can test if it satisfied in nature.

For example the SM satisfies finite naturalness (as discussed in section 2), and high-scale gauge unification does not satisfy it. While gauge unification is just a nice theoretical hypothesis, there are kinds of new physics which are demanded by experiments: neutrino masses

(discussed in section 3), Dark Matter (discussed in section 4), and possibly the strong CP problem (section 5) and vacuum stability and inflation (section 6). We will classify models of new physics from the point of view of finite naturalness, finding constraints on their parameter spaces which usually imply new particles not much above the weak scale. Conclusions are given in section 7.

2 The Standard Model

Here and in the rest of the paper we write the SM Higgs potential at tree level as

$$V = -\frac{m^2}{2}|H|^2 + \lambda |H|^4 \tag{3}$$

such that, expanding around the minimum of the Higgs doublet $H = (0, (v+h)/\sqrt{2})$ with $v = m/\sqrt{2\lambda} \approx 246.2\,\mathrm{GeV}$, the parameter m at tree level equals the physical Higgs mass, $M_h = \sqrt{2\lambda}v = m$.

The expectation is that finite naturalness is satisfied by the SM quantum corrections, because the top and the vectors are not much heavier than the Higgs. To confirm this we compute at one-loop accuracy the Higgs mass parameter m, in the $\overline{\mathrm{MS}}$ scheme. We find

$$m^2 = M_h^2 + \mathrm{Re}\,\Pi_{hh}^{(1)}(p^2 = M_h^2)|_{\rm finite} + 3\frac{T^{(1)}|_{\rm finite}}{v} \tag{4}$$

where M_h is the pole Higgs mass, $\Pi_{hh}^{(1)}(p^2 = M_h^2)$ is the on-shell one-particle-irreducible Higgs propagator and $T^{(1)}$ is the Higgs tadpole. Their sum reconstructs the full Higgs propagator. Computing them in a generic R_ξ gauge we find that their combination in eq. (4) is gauge-independent, as it should [7], and explicitly given by[1]

$$\begin{aligned}
m^2 &= M_h^2 + \frac{1}{(4\pi v)^2}\bigg[6M_t^2(M_h^2 - 4M_t^2)B_0(M_h; M_t, M_t) + 24M_t^2 A_0(M_t) + \\
&\quad +(M_h^4 - 4M_h^2 M_W^2 + 12M_W^4)B_0(M_h; M_W, M_W) - 2(M_h^2 + 6M_W^2)A_0(M_W) + \\
&\quad +\frac{1}{2}\left(M_h^4 - 4M_h^2 M_Z^2 + 12M_Z^4\right)B_0(M_h; M_Z, M_Z) - (M_h^2 + 6M_Z^2)A_0(M_Z) + \\
&\quad +\frac{9}{2}M_h^4 B_0(M_h; M_h, M_h) - 3M_h^2 A_0(M_h)\bigg] \qquad (5)\\
&= M_h^2\left(1 + 0.133 + \beta_m^{\rm SM}\ln\frac{\bar\mu^2}{M_t^2}\right) \qquad (6)
\end{aligned}$$

where

$$A_0(M) = M^2(1 - \ln\frac{M^2}{\bar\mu^2}), \qquad B_0(p; M, M) = -\int_0^1 \ln\frac{M^2 - x(1-x)p^2}{\bar\mu^2}dx \tag{7}$$

are the finite parts of the usual Passarino-Veltman functions and M_t is the top quark mass, M_W is the W mass, M_Z is the Z mass. This correction reproduces the well known one-loop SM RGE equation for m^2

$$\frac{dm^2}{d\ln\bar\mu^2} = \beta_m^{\rm SM} m^2, \qquad \beta_m^{\rm SM} = \frac{3}{4}\frac{4y_t^2 + 8\lambda - 3g^2 - g_Y^2}{(4\pi)^2}. \tag{8}$$

[1] As far as we know, this is the first computation with one-loop accuracy of the dimensional fundamental parameter of the SM.

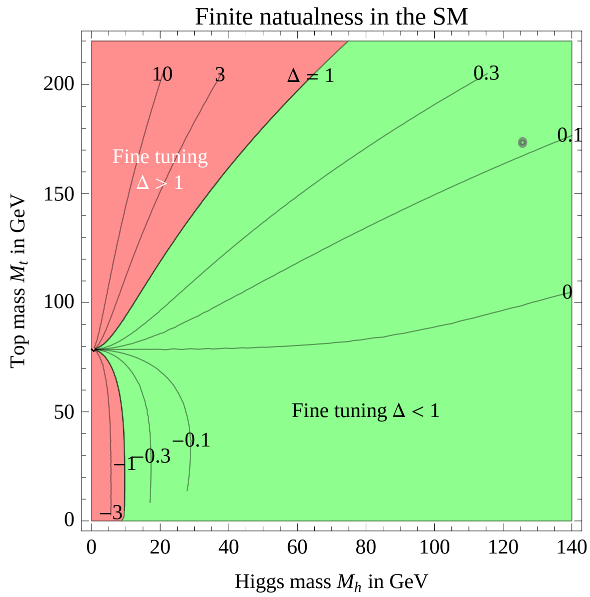

Figure 1: *The SM satisfies 'finite naturalness' for the observed values of its parameters (small ellipse), while a large fine-tuning would be present for a lighter Higgs.*

In view of the log divergence, the finite part of the correction to m^2 is scheme-dependent; it depends on the definition and on the value of $\bar{\mu}$. From eq. (6) we see that the $\overline{\rm MS}$ Higgs mass parameter equals $m(\bar{\mu} = M_t) = 132.7\,{\rm GeV}$. Renormalizing it at large energies [8] we find $m(\bar{\mu} = M_{\rm Pl}) = 140.9\,{\rm GeV}$. As a consequence the SM satisfies finite-naturalness, for the observed values of its parameters. Fig. 1 shows contour-levels of the fine-tuning $\Delta \equiv m^2(M_t)/M_h^2 - 1$: we see that $\Delta \approx 0.13$ is small for the observed values of the SM parameters, while a Higgs mass ≈ 10 lighter than the top would have led to a 'finite naturalness' problem within the SM.

We now explore the implications for 'finite naturalness' of new physics motivated by observations.

3 Finite naturalness, neutrino masses and leptogenesis

The observation of neutrino masses [9], presumably of Majorana type, points to new physics at some scale possibly as high as $v^2/m_\nu \sim 10^{14}\,{\rm GeV}$. At tree level, neutrino masses can be mediated by 3 types of new particles, called type I, II and III see-saw. We will study the corrections to the Higgs mass parameter in these scenarios.

3.1 Type-I see saw

Type-I see-saw contains heavy right-handed neutrinos N with mass M and Yukawa couplings $\lambda_N\ NLH$ to lepton doublets L such that at low energy one obtains Majorana neutrino masses

$m_\nu = \lambda_N^2 v^2/M$ [9]. The one-loop correction to the squared Higgs mass is

$$\delta m^2 = \frac{4\lambda_N^2}{(4\pi)^2} M^2 (\ln \frac{M^2}{\bar\mu^2} - 1) \tag{9}$$

where the RGE scale $\bar\mu$ can be identified with the cut-off of the theory, possibly the Planck scale. Indeed the log-enhanced term in eq. (9) means that, above M, heavy right-handed neutrinos produce the following term

$$\frac{dm^2}{d\ln\bar\mu^2} = \frac{4\lambda_N^2}{(4\pi)^2} M^2 + \beta_m^{\rm SM} m^2 \tag{10}$$

in the RGE for the Higgs mass term squared parameter m^2. Therefore the condition of finite naturaless, $\delta m_h^2 \lesssim m_h^2 \times \Delta$ (where Δ is an order-one fine-tuning factor), is satisfied by type-I models if right-handed neutrinos are lighter than [10]

$$M \lesssim m_h \left(\Delta \frac{16\pi^2 m_h}{m_\nu}\right)^{1/3} \approx 0.7\, 10^7\,{\rm GeV} \times \sqrt[3]{\Delta} \qquad \text{(Type-I see-saw)} \tag{11}$$

having assumed $m_\nu = (\Delta m^2_{\rm atm})^{1/2} \approx 0.05\,{\rm eV}$. This upper bound on M is hardly compatible with thermal leptogenesis, that needs $M \gtrsim 2\; 10^9\,{\rm GeV}$, unless right-handed neutrinos dominated the energy density of the universe [11] (such that leptogenesis can be successful for $M \gtrsim 2 \cdot 10^7\,{\rm GeV}$) and/or in presence of resonant enhancements.

It is interesting to discuss what happens if condition (11) is not satisfied. Then, m^2 must have at large energy a large positive value (that corresponds to a more unstable potential) in order to get the observed small $M_h \approx 125\,{\rm GeV}$ at low energy. Given that the quartic coupling λ becomes $\lambda \approx 0$ when renormalised at a scale around $10^7\,{\rm GeV}$, the one-loop potential might there develop new features (such as a new local minimum). We verified that this is not the case.

3.2 Type-III see saw

In type-III see-saw models the heavy singlets N are replaced by heavy weak triplets N^a with zero hypercharge [9]. Beyond a correction to m^2 similar to eq. (9), there is now a bigger correction induced at two loops by the SU(2)$_L$ gauge couplings of the triplet. Using the results for the generic two-loop potential [12] we obtain

$$\delta m^2 = \frac{g_2^4}{(4\pi)^4} M^2 (36 \ln \frac{M^2}{\bar\mu^2} - 6). \tag{12}$$

The log-enhanced term reproduces the RGE equation for m^2 (that we computed at 2 loops using [13]):

$$\frac{dm^2}{d\ln\bar\mu^2} = \frac{36 g_2^4}{(4\pi)^4} M^2 + \beta_m^{\rm SM} m^2. \tag{13}$$

Assuming that the cut-off of the theory is around the Planck mass we find that the condition of finite naturalness, $\delta m_h^2 \lesssim m_h^2 \times \Delta$, demands that the weak triplet must be lighter than

$$M \lesssim 0.94\,{\rm TeV} \times \sqrt{\Delta} \qquad \text{(Type-III see-saw)}. \tag{14}$$

Such a low mass is not compatible with successful thermal leptogenesis in the context of type-III see-saw [14, 15], but can lead to observable signals at LHC [16].

3.3 Type-II see saw

Type II see-saw employs one scalar T^a, triplet under $\mathrm{SU}(2)_L$ and with hypercharge $Y = 1$ [9]. Its relevant couplings are

$$\mathscr{L} = \mathscr{L}_{\rm SM} + |D_\mu T|^2 - M^2|T|^2 + \frac{1}{2}(\lambda_T^{ij} L^i \varepsilon \tau^a L^j T^a + \lambda_H M \; H\varepsilon\tau^a H\, T^{a*} + \text{h.c.}) - \lambda_{HT}|H|^2|T|^2 + \cdots \tag{15}$$

where λ_T is a symmetric flavour matrix, ε is the permutation matrix, and τ^a are the usual $\mathrm{SU}(2)_L$ Pauli matrices. Majorana neutrino masses arise as $m_\nu = \lambda_T \lambda_H v^2/M^2$. We ignore the corrections to m^2 due to non-minimal couplings non related to neutrino masses, except for λ_{HT}, because it will enter the subsequent discussion.

At one loop there is the correction to m^2 induced by λ_H:

$$\delta m^2 = -\frac{6\lambda_H^2 M^2}{(4\pi)^2}(\ln\frac{M^2}{\bar\mu^2} - 1). \tag{16}$$

At two loops there is the correction induced only by electroweak interactions:

$$\delta m^2 = -M^2 \frac{6g_2^4 + 3g_Y^4}{(4\pi)^4}(\frac{3}{2}\ln^2\frac{M^2}{\bar\mu^2} + 2\ln\frac{M^2}{\bar\mu^2} + \frac{7}{2}). \tag{17}$$

The $\ln^2\bar\mu$ term in eq. (17) arises as a composition of two one-loop effects: i) the coupling λ_{HT} is generated from pure gauge effects; ii) λ_{HT} affects the Higgs mass. These two effects are described by the following terms in the one loop RGE equations (which had previously been computed in [17]):

$$\frac{d\lambda_{HT}}{d\ln\bar\mu} = \frac{6g_2^4 + 3g_Y^4}{(4\pi)^2} + \cdots \qquad \frac{dm^2}{d\ln\bar\mu} = -\frac{12\lambda_{HT}}{(4\pi)^2}M^2 + \cdots \tag{18}$$

The $\ln\bar\mu$ term in eq. (17) arises from the finite parts in these one-loop effects, combined with the pure two-loop RGE for m^2

$$\frac{dm^2}{d\ln\bar\mu} = -10\frac{6g_2^4 + 3g_Y^4}{(4\pi)^4}M^2. \tag{19}$$

Assuming $\bar\mu \sim M_{\rm Pl}$, the two loop gauge correction to m^2 of eq. (17) imply the finite naturalness bound

$$M \lesssim 200\,\mathrm{GeV} \times \sqrt{\Delta} \qquad \text{(Type-II see-saw)} \tag{20}$$

which is comparable with present LHC bounds [18]. Then the one-loop correction of eq. (16) sets a weak upper bound on λ_H, which does not forbid the possibility that $T \to HH$ is the dominant decay channel. Leptogenesis is not possible at such a low value of M [19].

In conclusion, 'finite naturalness' and neutrino masses suggest that type III and especially type II see-saw allow for direct signals at LHC. However, the most plausible possibility seems type-I see-saw with leptogenesis around $10^7\,\mathrm{GeV}$.

4 Finite naturalness and Dark Matter

Dark Matter exists, and presumably it is some new fundamental particle. However, its mass is totally unknown, ranging from the galactic scale to the Planck scale. The hypothesis that DM

is a thermal relic suggests that DM might be around the weak scale. Even so, there is a huge number of viable DM models. Here we focus on some representative models that, unlike supersymmetric DM, are not motivated by the usual formulation of the hierarchy problem. We choose Minimal Dark Matter [20] as a representative of models where DM has electro-weak interactions, and the minimal scalar singlet [21] and fermion singlet [22] models as representatives of models where DM has no electro-weak interactions. Axion DM is considered in the subsequent axion section.

4.1 Minimal Dark Matter

Minimal Dark Matter (MDM) assumes that DM is the neutral component of just one electroweak multiplet which only has electroweak gauge interactions. As summarized in table 1, a few choices of the multiplets are possible, and in each case everything is predicted as function of the DM mass M, which can be fixed assuming that the thermal relic density equals the observed DM density [20].

Coming to the condition of finite naturalness, we compute the correction to the Higgs squared mass term which is produced by electroweak gauge interactions at two loops.

- For a generic fermionic multiplet with hypercharge Y and dimension n under $\mathrm{SU}(2)_L$ we find
$$\delta m^2 = \frac{cnM^2}{(4\pi)^4}\left(\frac{n^2-1}{4}g_2^4 + Y^2 g_Y^4\right)\left(6\ln\frac{M^2}{\bar{\mu}^2}-1\right) \tag{21}$$
where $c=1$ for Majorana fermions ($Y=0$ and odd n) and $c=2$ for Dirac fermions ($Y\neq 0$ and/or even n). For $n=3$ and $Y=0$ we recover the type-III see-saw result of eq. (12).

- For a scalar multiplet we find
$$\delta m^2 = -\frac{nM^2}{(4\pi)^4}\left(\frac{n^2-1}{4}g_2^4 + Y^2 g_Y^4\right)\left(\frac{3}{2}\ln^2\frac{M^2}{\bar{\mu}^2}+2\ln\frac{M^2}{\bar{\mu}^2}+\frac{7}{2}\right). \tag{22}$$
For $n=3$ and $Y=0$ we recover the type-II see-saw result of eq. (17).

We then show in table 1 the finite naturalness upper bounds on M for the various possible MDM multiplets. Furthermore, table 1 shows the predictions for the DM mass M suggested by the hypothesis that DM is a thermal relic with cosmological abundance

$$\Omega_{\rm DM}h^2 = 0.1187 \pm 0.0017 \text{ [23]}. \tag{23}$$

(Such results differ from the analogous table of [20] because M has been recomputed taking into account Sommerfeld effects [24], which lead to the change indicated by the arrows in table 1). We see that for $n \gtrsim 4$ finite naturalness is not compatible with the larger value of M suggested by thermal DM. We recall that the most motivated MDM candidate is a fermion with $n=5$ (or a scalar with $n=7$) because such particles are automatically stable: just like in the case of the proton the first possible source of DM decay comes from dimension-6 operators, and thereby are naturally suppressed by a large scale.

In table 1 we also recomputed the prediction for the spin-independent direct detection cross section $\sigma_{\rm SI}$. With respect to the analogous table of [20] we recomputed $\sigma_{\rm SI}$ taking into

Quantum numbers $\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	Spin	DM could decay into	DM mass in TeV	$m_{\mathrm{DM}^{\pm}} - m_{\mathrm{DM}}$ in MeV	Finite naturalness bound in TeV	σ_{SI} in $10^{-46}\,\mathrm{cm}^2$
2	1/2	0	EL	0.54	350	$0.4 \times \sqrt{\Delta}$	$(2.3 \pm 0.3)\,10^{-2}$
2	1/2	1/2	EH	1.1	341	$1.9 \times \sqrt{\Delta}$	$(2.5 \pm 0.8)\,10^{-2}$
3	0	0	HH^*	$2.0 \to 2.5$	166	$0.22 \times \sqrt{\Delta}$	0.60 ± 0.04
3	0	1/2	LH	$2.4 \to 2.7$	166	$1.0 \times \sqrt{\Delta}$	0.60 ± 0.04
3	1	0	HH, LL	$1.6 \to ?$	540	$0.22 \times \sqrt{\Delta}$	0.06 ± 0.02
3	1	1/2	LH	$1.9 \to ?$	526	$1.0 \times \sqrt{\Delta}$	0.06 ± 0.02
4	1/2	0	HHH^*	$2.4 \to ?$	353	$0.14 \times \sqrt{\Delta}$	1.7 ± 0.1
4	1/2	1/2	(LHH^*)	$2.4 \to ?$	347	$0.6 \times \sqrt{\Delta}$	1.7 ± 0.1
4	3/2	0	HHH	$2.9 \to ?$	729	$0.14 \times \sqrt{\Delta}$	0.08 ± 0.04
4	3/2	1/2	(LHH)	$2.6 \to ?$	712	$0.6 \times \sqrt{\Delta}$	0.08 ± 0.04
5	0	0	(HHH^*H^*)	$5.0 \to 9.4$	166	$0.10 \times \sqrt{\Delta}$	5.4 ± 0.4
5	0	1/2	stable	$4.4 \to 10$	166	$0.4 \times \sqrt{\Delta}$	5.4 ± 0.4
7	0	0	stable	$8 \to 25$	166	$0.06 \times \sqrt{\Delta}$	22 ± 2

Table 1: **Minimal Dark Matter**. *The first columns define the quantum numbers of the possible DM weak multiplets. Next we show the possible decay channels which need to be forbidden; the DM mass predicted from thermal abundance (the arrows indicate the effect of taking into account non-perturbative Sommerfeld corrections, which have not been computed in all cases); the predicted splitting between the charged and the neutral components of the DM weak multiplet; the bound from finite naturalness and the prediction for the Spin-Independent direct detection cross section on protons* σ_{SI}.

account: a) the correct value of the Higgs mass, which has now been measured; b) two-loop DM/gluon interactions which partially cancel the one-loop DM/quark interactions [25]; c) improved determinations of the nucleon matrix elements. In particular new experimental measurements of πp scattering and new lattice simulations point to a strange content of the nucleon $f_s \equiv \langle N|m_s \bar{s}s|N\rangle/m_N = 0.043 \pm 0.011$ [26], lower than the value $f_s \sim 0.3$ previously assumed. All these effect reduce the predicted value of σ_{SI}. We refer to table 1 of [27] for all other nuclear inputs. Following [20], for the scalar MDM candidates we neglected the contribution to σ_{SI} from a possible quartic coupling between DM and the Higgs. Furthermore, pure MDM candidates with $Y \neq 0$ are excluded by a too large tree level contribution to σ_{SI} from Z exchange. We assumed that such effect is removed by adding a small mass mixing between the real and imaginary component of the neutral component, such that DM is a real particle and only the loop contribution is present [20].

The MDM weak multiplets present at low energy change the running of g_2, g_Y such that the SM couplings renormalised at large energy can satisfy the Veltman throat condition at the Planck scale (this is roughly achieved by adding a doublet or a $Y = 0$ triplet) or give absolute stability of the Higgs potential (this needs larger corrections to the β-functions).

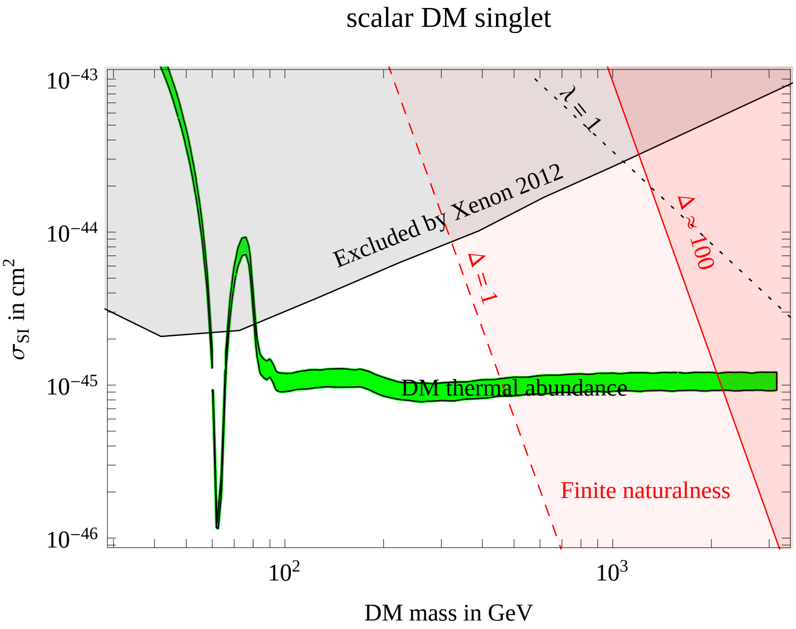

Figure 2: **Scalar singlet DM model**. *In the $(M, \sigma_{\rm SI})$ plane we plot the region excluded by the Xenon direct search experiment, the band favored by the thermal DM abundance, the upper bound on the DM mass following from finite naturalness for $\Delta = 1$ and $\Delta = 100$.*

4.2 The scalar singlet DM model

We now consider DM without SM gauge interactions. A scalar singlet S that respects a $S \to -S$ symmetry is added to the SM, such that S is a stable DM candidate. The model is described by the Lagrangian [21]

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \frac{(\partial_\mu S)^2}{2} - \frac{m_S^2}{2} S^2 - \lambda_{HS} S^2 |H|^2 - \frac{\lambda_S}{4} S^4 \ . \tag{24}$$

The DM mass is given by $M^2 = m_S^2 + 2\lambda_{HS} v^2$. The quantum correction to the Higgs mass parameter m^2 is

$$\delta m^2 = -\frac{2\lambda_{HS}^2}{(4\pi)^2} M^2 (\ln \frac{M^2}{\bar\mu^2} - 1) \qquad \text{i.e.} \qquad \frac{dm^2}{d\ln\bar\mu^2} = -\frac{2\lambda_{HS}^2}{(4\pi)^2} M^2 + \beta_m^{\rm SM} m^2 \tag{25}$$

such that the finite naturalness bound on the DM mass depends on the unknown Higgs/DM coupling λ_{HS}. This coupling can be fixed by DM physics. Indeed, the spin-independent cross section relevant for direct DM detection is predicted as [21]

$$\sigma_{\rm SI} = \frac{\lambda_{HS}^2 m_N^4 f^2}{\pi M^2 m_h^4} \tag{26}$$

where $f \approx 0.295$ is the nucleon matrix element. In fig. 2 we plot the finite naturalness bound in the $(M, \sigma_{\rm SI})$ plane. Imposing $\delta m^2 \lesssim m_h^2 \times \Delta$ for a running from the Planck scale down to the weak scale we find that no fine-tuning needs DM to be lighter than a few hundred GeV (red dashed curve), which corresponds to a coupling $\lambda_{HS} \lesssim 0.1$. By either allowing for a $\Delta \approx 100$ fine-tuning, or by dropping the $\ln(M_{\rm Pl}^2/M^2) \approx 80$ RGE enhancement in the loop effect, gives the weaker bound represented by the continuous red curve. This bound is stronger than

the one obtained by demanding perturbativity of λ_{HS} (exemplified by the dotted curve that corresponds to $\lambda_{HS} = 1$).

Furthermore, in the same plot we show the region (green band) where λ_{HS} is fixed by the assumption that the thermal relic DM density reproduces the measured DM density $\Omega_{\rm DM}$, as computed in tree-level approximation [21].

We incidentally observe that quadratic divergences to m^2 could vanish because of a Veltman throat at the Planck scale: this requires $\lambda_{HS}(M_{\rm Pl}) \approx 0.2$ having assumed the SM values for all other couplings.

4.3 The fermion singlet model

We here consider a model where DM is a fermionic singlet ψ with no SM gauge interactions. Given that a fermion singlet cannot have direct renormalizable couplings to SM particles, one needs to add something else and many possibilities arise. For example one can add an inert Higgs doublet H' allowing for the coupling $\psi H' L$: then the weak gauge interactions of H' would produce the 'finite naturalness' bound computed in eq. (22). As we here want to explore DM systems without SM gauge interactions, we add a neutral scalar singlet S, such that the Lagrangian is [22]

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \frac{(\partial_\mu S)^2}{2} + \bar\psi i\partial\!\!\!/\psi - \frac{m_S^2}{2}S^2 - \frac{\lambda_S}{4}S^4 - \lambda_{HS}S^2|H|^2 + \frac{y}{2}S\psi\psi + \frac{M_\psi}{2}\psi\psi + \text{h.c.} \tag{27}$$

We consider the minimum where both the Higgs and S get a vacuum expectation value:

$$v^2 = \frac{\lambda_S m^2 + 2\lambda_{HS}m_S^2}{4(\lambda\lambda_S - \lambda_{HS}^2)}, \qquad \langle S\rangle^2 = \frac{4\lambda m_S^2 + 2\lambda_{HS}m^2}{4(\lambda_{HS}^2 - \lambda_S\lambda)}. \tag{28}$$

The mixing among h and S leads to scalar mass eigenstates S_1 and S_2:

$$S = \cos\alpha\, S_2 - \sin\alpha\, S_1\,, \qquad h = \sin\alpha\, S_2 + \cos\alpha\, S_1 \tag{29}$$

where the mixing angle α is

$$\tan 2\alpha = \frac{4\sqrt{2}\lambda_{HS}v\langle S\rangle}{\lambda v^2 - 2\lambda_S\langle S\rangle^2}. \tag{30}$$

We identify S_1 with the 125 GeV Higgs field. The one-loop correction to the Higgs squared mass is then

$$\delta m^2 = \frac{6y^2\sin^2\alpha}{(4\pi)^2}M^2(\ln\frac{M^2}{\bar\mu^2} - \frac{1}{3}) \tag{31}$$

where $M = M_\psi + y\langle S\rangle$ is the DM mass. Its spin-independent cross section on nucleons is given by [22]

$$\sigma_{\rm SI} = \frac{y^2\sin^2 2\alpha}{8\pi}\frac{m_N^4 f^2}{v^2}\left(\frac{1}{m_1^2} - \frac{1}{m_2^2}\right)^2. \tag{32}$$

The DM annihilation cross section, at tree level, is given by [22]

$$\sigma(\psi\psi\to X) = \sigma(\psi\psi\to \text{SM}) + \sum_{i,j=1,2}\sigma(\psi\psi\to S_iS_j). \tag{33}$$

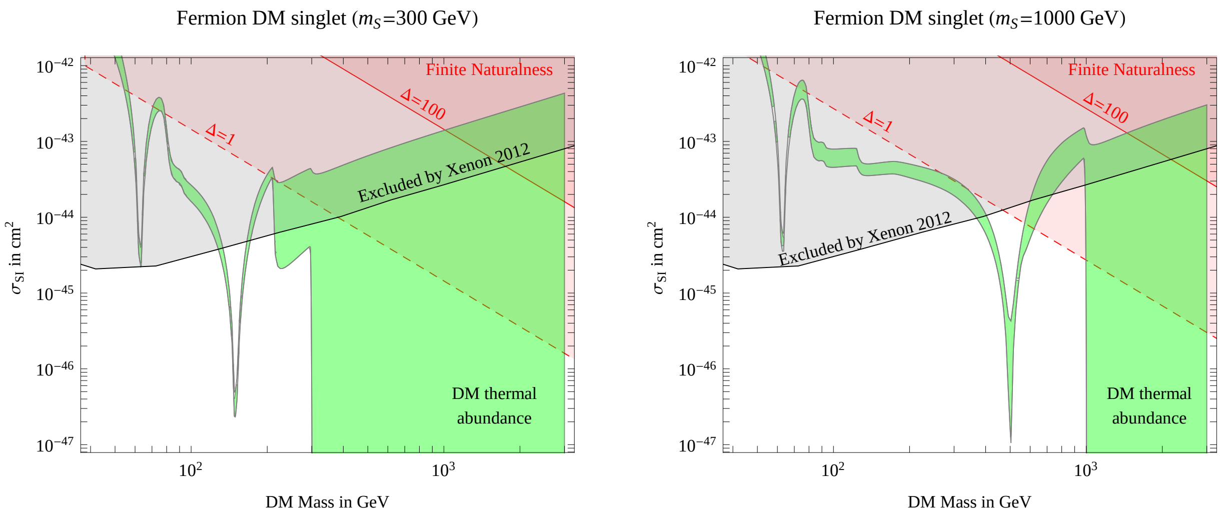

Figure 3: **Fermion singlet DM model**. *In the* $(M, \sigma_{\rm SI})$ *plane we plot the region excluded by the Xenon direct search experiment and the upper bound on the DM mass following from finite naturalness for fine-tuning* $\Delta = 1$ *(dashed line) and* $\Delta = 100$ *(continuous line). We assume* $m_{S_2} = 300\,{\rm GeV}$ *(left) or* $1\,{\rm TeV}$ *(right). This does not fix all parameters of the model such that the region where the DM thermal density can reproduce the cosmological DM density (green band) becomes wide at* $M > m_{S_2}$*. We demand compatibility with electroweak precision data.*

Both terms are p-wave suppressed. The first term includes all annihilations into SM particles mediated by s-channel exchange of one of the two scalars S_i:

$$\sigma(\psi\psi \to {\rm SM}) = v_{rel} \frac{y^2 M^2 \sin^2 2\alpha}{4} \frac{\Gamma_{h\to{\rm SM}}(2M)}{2M} \left| \frac{i}{4M^2 - m_1^2 + i m_1 \Gamma_{S_1}} - (1 \leftrightarrow 2) \right|^2 \tag{34}$$

where v_{rel} is the relative DM velocity, $\Gamma_{h\to{\rm SM}}(m)$ is the width of a SM Higgs of mass m and Γ_{S_i} are the widths of the two physical scalars in the model. This contribution vanishes in absence of mixing, namely when $\langle S\rangle = 0$.

The second term in eq. (33) receives two kinds of contributions: s-channel exchange of the two scalars and t and u-channel exchange of ψ. The former process is proportional to the quartic coupling λ_{HS} which we assume to be small, while the latter is non vanishing also in the absence of mixing between S and h. It is given by

$$\sigma(\psi\psi \to S_i S_j) = v_{rel} \frac{3y^4 c_i^2 c_j^2 f_{ij}}{64\pi M^2 (1+\delta_{ij})} \tag{35}$$

where $c_1 = \sin\alpha$, $c_2 = \cos\alpha$ and

$$f_{ij} = \frac{2304M^8 - 1024M^6(m_i^2+m_j^2) + 32M^4(3m_i^2+m_j^2)(m_i^2+3m_j^2) + (m_i^2-m_j^2)^4}{9(4M^2-m_i^2-m_j^2)^4} \sqrt{1 + \frac{(m_i^2-m_j^2)^2}{(2M)^4} - \frac{2(m_i^2+m_j^2)}{(2M)^2}} \tag{36}$$

normalized to satisfy $f_{ij} = 1$ for $m_{i,j} \to 0$.

The number of free parameters (five) of the model and the variety of possible resonant enhancements make it difficult to analyze it. In fig. 3 we consider the $(M, \sigma_{\rm SI})$ plane relevant

for direct DM searches. We also fix m_{S_2} such that resonances arise at fixed values of the DM mass M, and such that the quantum correction to the Higgs mass, eq. (31), is determined in terms of $M, \sigma_{\rm SI}$ and m_{S_2}. This allows us to computed the 'finite naturalness' bound, plotted in fig. 3. Furthermore, we plot the range (in green) where the cosmological DM abundance can be reproduced thermally; in view of the extra undetermined parameters such band becomes wide at $M > m_{S_2}$. Its sharp structure corresponds to the various possible resonances. Such band has been restricted imposing that Higgs data and electroweak precision tests are satisfied.[2] We see that DM is constrained by 'finite naturalness' to be lighter than a few TeV in the region where direct detection experiments can test this model.

5 Finite naturalness and axions

Axions are a solution to the strong CP problem [28] that requires new physics coupled to the SM at a scale f_a which is experimentally constrained to be $f_a \gtrsim 10^9\,\mathrm{GeV}$, much above the weak scale [29]. Furthermore, axions can be DM and provide the observed DM density via the 'misalignment' production mechanism provided that $f_a \approx 10^{11}\,\mathrm{GeV}$, or higher in presence of fine-tunings in the phase of the initial axion vacuum expectation value [30].

So, the main issue is if these larger scales mean that finite naturalness is unavoidably violated. As we will see, the answer is no. To address this issue we separately analyze the two main classes of axion models.

5.1 KSVZ axions

KSVZ axion models [31] employ new heavy quarks Ψ whose mass arises only from the vev of a complex axion scalar A that spontaneously breaks a global $\mathrm{U}(1)_{\rm PQ}$ symmetry:

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \bar\Psi i\partial\!\!\!/\Psi + |\partial_\mu A|^2 + (\lambda_{A\Psi} A\Psi_L\Psi_R + \text{h.c.}) + V(A) + \lambda_{AH}|AH|^2 \tag{40}$$

The light axion a is the phase of A. Integrating out the heavy quarks Ψ gives rise to the usual anomalous couplings of the light axion suppressed by f_a, even if the heavy quark mass $M = \lambda_{A\Psi}\langle A\rangle = \lambda_{A\Psi} f_a e^{ia/f_a}$ is much smaller than f_a.

Finite naturalness holds if the coupling λ_{AH} is smaller than about $(4\pi v/f_a)^2$. This coupling is unavoidably generated from RGE at two loops as $\delta\lambda_{AH} \sim g^2\lambda_{AQ}^2/(4\pi)^4$. This effect is equivalent to computing the two loop correction from heavy fermions Ψ to the Higgs mass

[2] The mixing between S and the Higgs corrects the electroweak parameters as

$$\hat T = \frac{3g_2^2}{64\pi^2}\left[\sin^2\alpha\left(f_T(w_2) - \frac{f_T(z_2)}{c_W^2}\right) - \cos^2\alpha\left(f_T(w_1) - \frac{f_T(z_1)}{c_W^2}\right)\right], \tag{37}$$

$$\hat S = \frac{g_2^2}{32\pi^2}\left[\sin^2\alpha\, f_S(z_2) - \cos^2\alpha\, f_S(z_1)\right], \tag{38}$$

where $z_i = m_i^2/m_Z^2$ and $w_i = m_i^2/m_W^2$. The functions f_S and f_T are given by

$$\begin{aligned} f_S(x) &= \frac{1}{12}\left[-2x^2 + 9x + \left(x^2 - 6x + 18\frac{x-2}{x-1}\right)x\log x + 2\sqrt{x^2-4x}(x^2-4x+12)g(x)\right] \\ f_T(x) &= \frac{x\log x}{x-1}, \qquad g(x) = \begin{cases} \tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x-4}}\right) - \tan^{-1}\left(\frac{x-2}{\sqrt{x^2-4x}}\right) & \text{if } 0<x<4 \\ \frac{1}{2}\log\frac{1}{2}\left(x-2-\sqrt{x^2-4x}\right) & \text{if } x>4 \end{cases}. \end{aligned} \tag{39}$$

term, which is precisely given by eq. (21) times the colour multiplicity of Ψ (which is 3 for colour triplets). Then, finite naturalness implies the following upper bounds on the mass of the heavy fermions

$$M \lesssim \sqrt{\Delta} \times \begin{cases} 0.74\,\mathrm{TeV} & \text{if } \Psi = Q \oplus \bar{Q} \\ 4.5\,\mathrm{TeV} & \text{if } \Psi = U \oplus \bar{U} \\ 9.1\,\mathrm{TeV} & \text{if } \Psi = D \oplus \bar{D} \end{cases} \tag{41}$$

having assumed that they have the same quantum numbers as the SM quarks usually denoted as Q, U and D. Furthermore, there is a 3-loop RGE effect which applies even if Ψ has only strong interactions and no electroweak interactions: the heavy quarks Ψ interact with gluons, that interact with the top quark, that interacts with the Higgs, so that

$$\delta m^2 \sim \frac{g_3^4 y_t^2}{(4\pi)^6} M^2 \ln \frac{M^2}{\bar{\mu}^2}. \tag{42}$$

This correction implies a finite naturalness bound on $M \lesssim \sqrt{\Delta} \times 10\,\mathrm{TeV}$. With multiple heavy fermions, the bounds in (41) apply to the heaviest one.

In conclusion, the expectation from finite naturalness and KSVZ axions is heavy coloured new fermions with a mass around the TeV scale. In the absence of Yukawa couplings $H\psi\Psi$ (between the Higgs H, the SM quarks ψ and the heavy quarks Ψ) the latter would appear at LHC as stable heavy hadrons; their cosmological thermal relic abundance would be suppressed by the strong annihilation cross section. If $H\psi\Psi$ is present, Ψ would behave as heavy quarks; the interactions of such heavy quarks in the thermal plasma of the early universe would lead to a population of axions with $T_a \approx 0.903\,\mathrm{K}$. If $m_a \ll T_a$ this corresponds to extra radiation equivalent to $\Delta N_\nu \approx 0.026$ effective neutrinos, compatibly with existing data [23].

5.2 DFSZ axions

DFSZ axion models [32] employ two Higgs doublets H_u and H_d and a complex scalar A without SM gauge interactions such that the Lagrangian

$$\mathscr{L} \supset (\lambda_U\, UQH_u + \lambda_D DQH_d + \lambda_{AH} A^2 H_u H_d + \text{h.c.}) + V(A) + V(H_u, H_d). \tag{43}$$

has a global $\mathrm{U}(1)_{\rm PQ}$ symmetry broken by the vacuum expectation value of A. Since the SM quarks Q, U, D have non zero charges under the PQ symmetry, they generate the QCD-QCD-PQ anomaly. The condition of finite naturalness arises already at tree level, $\delta m^2 \sim \lambda_{AH} f_a^2$ and demands a very small $\lambda_{AH} \lesssim (v/f_a)^2 \lesssim 10^{-15}$.

6 Finite naturalness, vacuum decay and inflation

Another experimental result which might suggest the presence of physics beyond the SM is the fact that the SM potential (for the currently favored values of the Higgs and top masses) develops an instability at field values above about $10^8\,\mathrm{GeV}$, leading to vacuum decay with a rate much longer than the age of the universe [6].

There are many ways to avoid this instability, which employ loop corrections from new particles with sizeable couplings to the Higgs [33]. Thereby, in the context of finite naturalness, this kind of new physics is expected to be around the weak scale.

This is however not a general conclusion. Indeed there is one special model where the instability is avoided by a tree level effect with small couplings. Adding to the SM a scalar singlet S with interactions to the Higgs described by the potential [34]

$$V = \lambda_H \left(H^\dagger H - v^2\right)^2 + \lambda_S \left(S^\dagger S - w^2\right)^2 + 2\lambda_{HS} \left(H^\dagger H - v^2\right)\left(S^\dagger S - w^2\right) \tag{44}$$

the low-energy Higgs quartic coupling is given by $\lambda = \lambda_H - \lambda_{HS}^2/\lambda_S$ at tree level. This model allows to stabilize the SM vacuum compatibly with 'finite naturalness' even if the singlet is much above the weak scale, provided that the couplings λ_{HS} and λ_S are small. A singlet with this kind of couplings is present within an attempt of deriving the SM from the framework of non commutative geometry [35].

Finally, observations of cosmological inhomogeneities suggest that the full theory incorporates some mechanism for inflation. At the moment the connection with the SM is unknown, even at a speculative level. A successful inflaton must have a flat potential, which is difficult to achieve in models; at quantum level flatness usually demands small couplings of the inflaton to SM particles. An inflaton decoupled from the SM would satisfy 'finite naturalness'. A free scalar S with mass $M \approx 10^{13}\,\mathrm{GeV}$ is the simplest inflaton candidate; it satisfies finite naturalness provided that its couplings to the Higgs λ_{HS} is smaller than about 10^{-10}. A model of inflation at the weak scale based on a singlet S was proposed in [36] and anyhow needs a small coupling to the Higgs to preserve flatness.

7 Conclusions

Experiments are clarifying whether the smallness of the Higgs mass respects naturalness or not. Understanding the origin of the weak scale is particularly important from a meta-physical point of view, because here is where two different views of the universe clash: do we live in a 'good' supersymmetric universe with no tunings, or in a 'bad' multiverse where huge tunings make some of its regions anthropically observable?

We here explored the 'ugly' third possibility: that the usual formulation of naturalness [1] must be modified by ignoring the uncomputable quadratically divergent corrections to the squared Higgs mass, and keeping only the finite computable corrections.

The usual naturalness is not satisfied by the SM, and demands supersymmetry, or some other big modification of particle physics. Such new physics must be around the weak scale or at most a one-loop factor above it, as computed in eq. (2) for the coloured stops.

The modified 'finite naturalness' is satisfied by the SM for the measured values of its parameters, as we have shown by computing the Higgs mass parameter with one-loop accuracy.

However, experimental data demand new physics. The consequent extension of the Standard Model can satisfy 'finite naturalness' provided that such new physics is not much above the weak scale. The connection with the weak scale often arises at two-loops, such that 'finite naturalness' is not yet challenged by negative results of experimental searches. We find that:

- **Neutrino masses** can be mediated at tree level by three kinds of see-saw models, that employ new particles with mass M. We find the following finite naturalness bounds:

$$M \lesssim \left\{ \begin{array}{ll} 0.7\,10^7\,\mathrm{GeV} \times \sqrt[3]{\Delta} & \text{type I see-saw model,} \\ 200\,\mathrm{GeV} \times \sqrt{\Delta} & \text{type II see-saw model,} \\ 940\,\mathrm{GeV} \times \sqrt{\Delta} & \text{type III see-saw model,} \end{array} \right. \tag{45}$$

where $\Delta \sim 1$ is a fine-tuning factor. Thermal leptogenesis seems possible only in the first case.

- **Dark Matter** might a new particle. Assuming that DM has weak interactions, we computed the 'finite naturalness' bound on its mass M summarizing the results in table 1. The weakest bound, $M \lesssim 1.9\,\mathrm{TeV} \times \sqrt{\Delta}$ is obtained for a fermion doublet. Stronger bounds are obtained for scalar DM (e.g. $M \lesssim 0.4\,\mathrm{TeV} \times \sqrt{\Delta}$ for a scalar doublet) and for bigger $\mathrm{SU}(2)_L$ multiplets (e.g. $M \lesssim 1.0\,\mathrm{TeV} \times \sqrt{\Delta}$ for a fermion triplet and $M \lesssim 0.4\,\mathrm{TeV} \times \sqrt{\Delta}$ for a fermion quintuplet). We also considered models where DM has no SM gauge interactions and couples to the Higgs doublet, finding bounds at the TeV level under the assumption that DM is a thermal relic and/or that direct detection is detectable.

- **Axions** can solve the strong CP problem and can be DM provided that they are associated with a new high scale, $f_a \sim 10^{11}\,\mathrm{GeV}$. We have shown that axion models can be compatible with 'finite naturalness'; within KSVZ axion moles compatibility implies new quarks at the TeV scale.

- **Vacuum decay and inflation** do not lead to concrete restrictions.

In conclusion, extensions of the SM motivated by observed phenomena can be compatible with 'finite naturalness', and this often demands new particles not much above the weak scale and potentially accessible to colliders.

The smallness of the cosmological constant seems to violate both versions of naturalness.

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